**Coefficient of Discharge**

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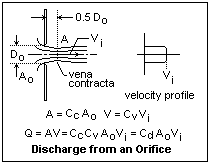
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**Theory of Discharge from an Orifice**

The Roman engineer Frontinus, who was in charge of the water supply under Augustus, used short pipes of graduated sizes to meter water delivered to different users. This was purely empirical, since the effects of pressure, or "head," and orifice size were not known quantitatively until Torricelli, in 1643, showed that the velocity of efflux was given by Vi = √2gh. We still calculate the velocity from Bernoulli's principle, that h + p/ρg + V2/2, is a constant along a streamline in irrotational flow, which is equivalent to the conservation of energy.

We'll consider here the case of zero initial velocity, as at the surface of a liquid in a container with an orifice in the side. We assume that a streamline starts at the surface, a distance h above the orifice, and neglect the pressure on the surface of the liquid, since it would cancel out anyway. The streamline then leads somehow to the orifice, and out into the jet that issues from it. We choose the point at which the streamlines are parallel a short distance from the orifice, and find that the velocity there is Vi = √2gh, as given by Torricelli's theorem.

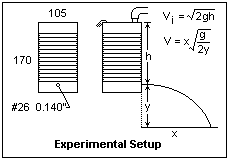
A jet surrounded only by air (or another fluid of small density) is called a *free jet*, and is acted upon by gravity. A jet surrounded by fluid is called a *submerged jet*. If the fluid is the same as that of the jet, then buoyancy eliminates the effect of gravity on it. A submerged jet is also subject to much greater friction at its boundary. We shall consider here only free jets of water, and neglect the viscosity of water, which is small, but finite.

A cross section of a circular orifice of diameter Do is shown. The thickness of the wall is assumed small compared to the diameter of the orifice. Because of the convergence of the streamlines approaching the orifice, the cross section of the jet decreases slightly until the pressure is equalized over the cross-section, and the velocity profile is nearly rectangular. This point of minimum area is called the *vena contracta*. Beyond the vena contracta, friction with the fluid outside the jet (air) slows it down, and the cross section increases perforce. This divergence is usually quite small, and the jet is nearly cylindrical with a constant velocity. The jet is held together by surface tension, of course, which has a stronger effect the smaller the diameter of the jet.

The area A of the vena contracta is smaller than the area Ao of the orifice because the velocity is higher there (converging streamlines). For a sharp-edged, or "ideal" circular orifice, A/Ao = Cc = π/(π + 2) = 0.611. Cc is called the *coefficient of contraction*. For a sharp orifice, is usually estimated to be 0.62, a figure that can be used if the exact value is not known. For an orifice that resembles a short tube, Cc = 1, but then there are turbulence losses that affect the discharge.

The average velocity V is defined so that it gives the correct rate of discharge when it is assumed constant over the vena contracta, or Q = VA. Then, we can write V = CvVi, where Cv is the coefficient of velocity. The coefficient of velocity is usually quite high, between 0.95 and 0.99. Combining the results of this paragraph and the preceding one, the discharge Q = VA = CvViCcAo = CdAoVi. Cd, the coefficient of discharge, allows us to use the ideal velocity and the orifice area in calculating the discharge.

**Experiments**

Our apparatus consists of a tomato juice can with the top removed, and a hole near the bottom. With this can, a scale, and a timing device, we can measure the coefficients of discharge and velocity, and from them the coefficient of contraction, for the orifice. In this small-scale experiment, the influence of surface tension, adhesion and capillarity will be greater than for most practical cases. Nevertheless, we shall obtain very instructive results at small expense and in little time.

The first experiment, to measure Cd, is performed by measuring the time required for the container to empty between levels h1 and h2 through the orifice. To find the rate at which the level h decreases, we equate the rate at which fluid is leaving the reservoir to the discharge of the orifice: Acdh/dt = - CdAo√2gh, or dh/dt = -Kh1/2, where K = √(2g)Cd(Ao/Ac), or dh/h1/2 = -Kdt. Ac is the area of the cross-section of the can. Integrating and substituting the limits, we find 2(√h1 - √h2) = KT, where T is the time required.

The corrugations in the can make convenient reference points for the liquid level. For my experiment, Ao = 0.09932 cm2, Ac = 84.95 cm2, h1 = 10.5 cm, and h2 = 1.5 cm. Using these numbers, K = 0.0518Cd, and the difference in the square roots of the heights is 2.239, so T = 86.53/Cd seconds. The time was measured with my HP-48G calculator as 126.3 s. The result was Cd = 0.685.

The second experiment measures Cv. Water was allowed to run from the tap into the reservoir, keeping h constant at 16 cm. The height of the orifice was y = 10.0 cm, and the horizontal distance was 22.8 cm. Since x = vt and y = gt2/2, v = x√(g/2y). From this equation, V = 160 cm/s. By Torricelli's theorem, Vi = √(2gh) = 177 cm/s. Therefore, the coefficient of velocity was Cv = 160/177 = 0.90. Finally, the coefficient of contraction was 0.685/0.90 = 0.76. These figures are quite reasonable, if somewhat different from those for larger scales. Orifices are considered small if they have diameters less than 2.5 in and heads less that 3 ft, so these experiments are indeed small-scale, where effects that can be neglected on larger scales may play a role. These effects generally act to increase the coefficient of contraction, so our value is not out of line.

Other experiments and demonstrations suggest themselves. The discharge coefficient could also be found by keeping the head constant and measuring the water discharged in a known time interval. Orifices at different heights could be made to flow simultaneously, demonstrating the increase in velocity with head. Three holes would be appropriate. They should not be in the same vertical line, so that each jet would be independent. A square orifice could be considered. Care should be taken to make the orifice as accurate as possible. A short tube could be soldered at the hole. In particular, a *Borda's mouthpiece*, which is a short tube of length about equal to the radius of the orifice that projects into the reservoir. The ideal coefficient of contraction for a Borda's mouthpiece is 0.5. Flow separation should occur at the inner edge. These suggestions would require a little more preparation, but the experiments would still be very inexpensive and easy.